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# Title: Modelo matemático completo para Robots Seriales

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# Índice.

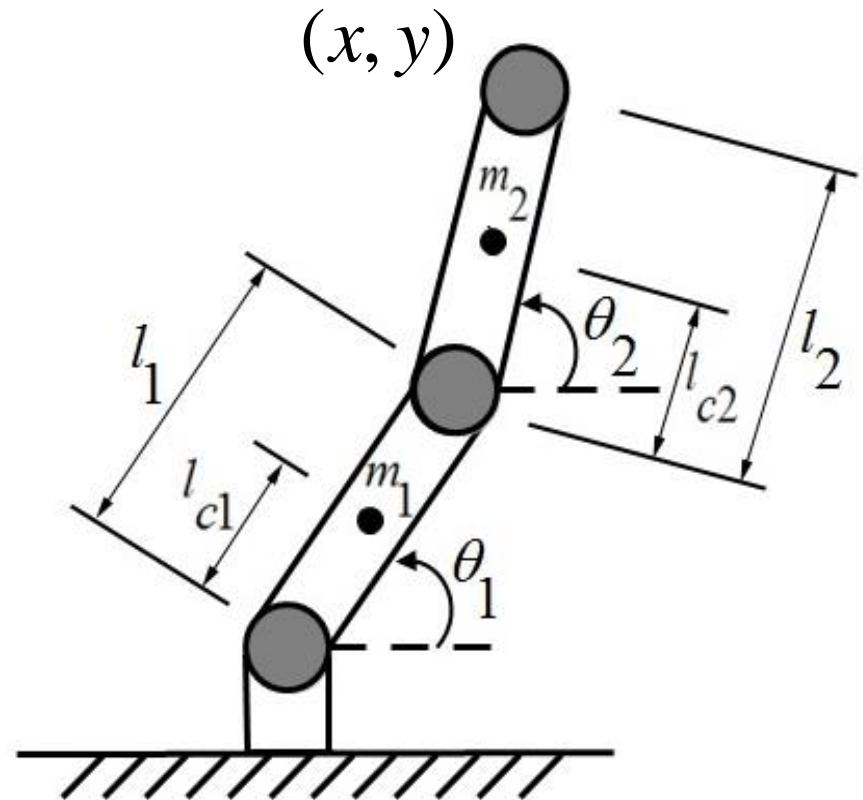
1. Cinemática directa fase simple.
2. Cinemática inversa fase simple.
3. Cinemática directa fase cerrada.
4. Cinemática inversa fase cerrada.
5. Dinámica fase simple.
6. Dinámica fase cerrada.



# Cinemática directa fase simple

$$x = l_1 \cos \theta_1 + l_2 \cos \theta_2$$

$$y = l_1 \sin \theta_1 + l_2 \sin \theta_2$$





# Cinemática Inversa fase simple

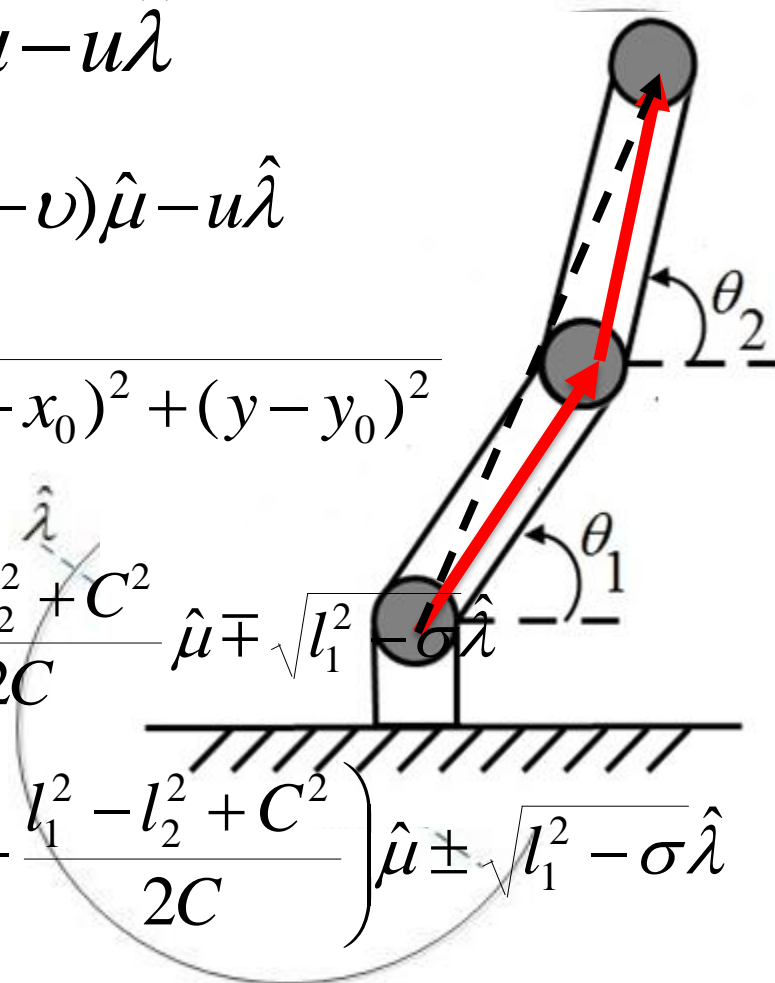
$$\hat{l}_1 = v\hat{\mu} - u\hat{\lambda}$$

$$\hat{l}_2 = (C - v)\hat{\mu} - u\hat{\lambda}$$

$$C = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

$$\hat{l}_1 = \frac{l_1^2 - l_2^2 + C^2}{2C} \hat{\mu} \mp \sqrt{l_1^2 - \sigma \hat{\lambda}}$$

$$\hat{l}_2 = \left( C - \frac{l_1^2 - l_2^2 + C^2}{2C} \right) \hat{\mu} \pm \sqrt{l_1^2 - \sigma \hat{\lambda}}$$





# Cinemática inversa fase simple

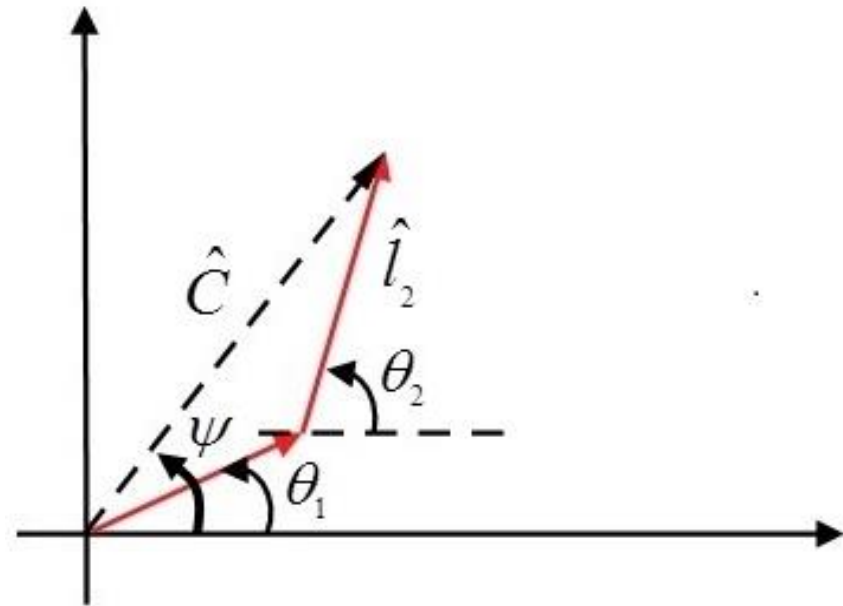
$$\hat{\mu} = [\|\hat{\mu}\| \cos \psi \quad \|\hat{\mu}\| \sin \psi \quad 0]^T = [\cos \psi \quad \sin \psi \quad 0]^T$$

$$\hat{\lambda} = \hat{\mu} \times \hat{k} \quad \hat{k} = [0 \quad 0 \quad 1]^T$$

$$\hat{\lambda} = [\sin \psi \quad -\cos \psi \quad 0]^T$$

$$\begin{bmatrix} l_{1x} \\ l_{1y} \end{bmatrix} = \frac{l_1^2 - l_2^2 + C^2}{2C} \begin{bmatrix} \cos \psi \\ \sin \psi \end{bmatrix} \mp \sqrt{l_1^2 - \sigma} \begin{bmatrix} \sin \psi \\ -\cos \psi \end{bmatrix}$$

$$\begin{bmatrix} l_{2x} \\ l_{2y} \end{bmatrix} = \left( C - \frac{l_1^2 - l_2^2 + C^2}{2C} \right) \begin{bmatrix} \cos \psi \\ \sin \psi \end{bmatrix} \mp \sqrt{l_1^2 - \sigma} \begin{bmatrix} \sin \psi \\ -\cos \psi \end{bmatrix}$$

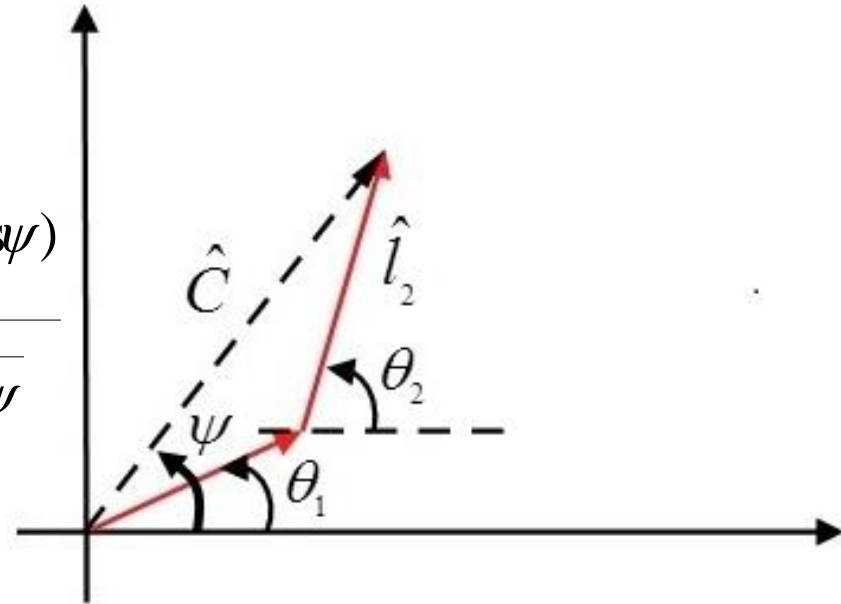




# Cinemática inversa fase simple.

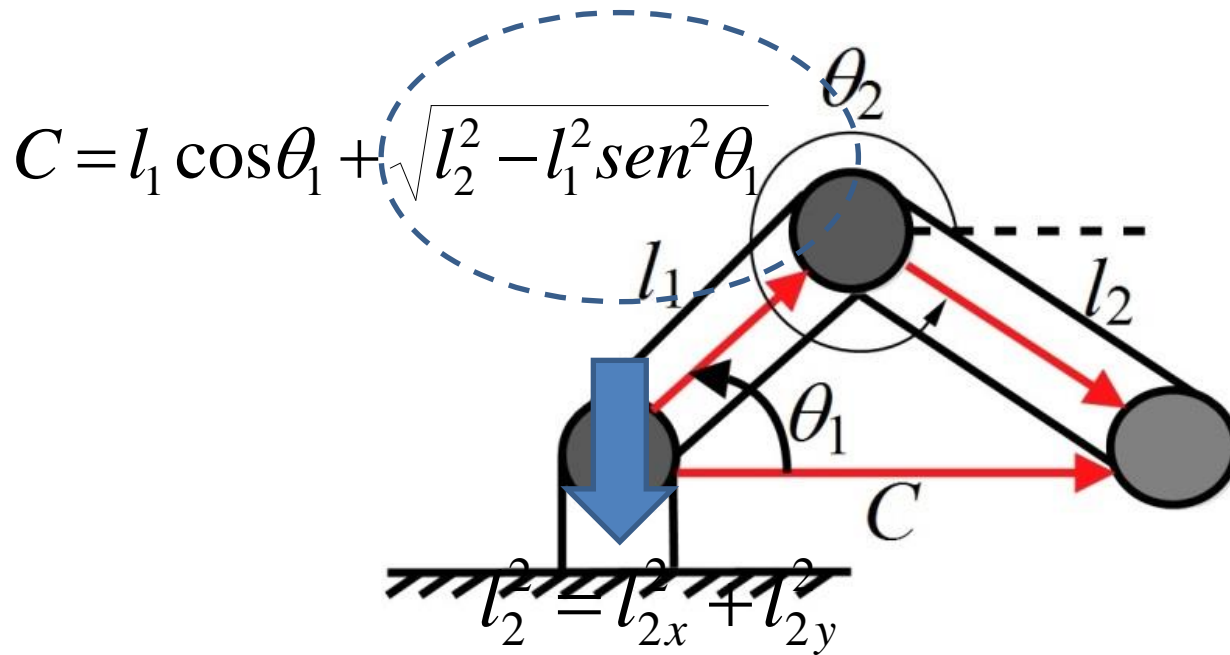
$$\begin{bmatrix} l_{px} \\ l_{py} \end{bmatrix} = \begin{pmatrix} l_1^2 - l_2^2 + C^2 + C^2 \cos \psi \\ 2C \sin \psi \end{pmatrix} \begin{bmatrix} \cos \psi \\ \sin \psi \end{bmatrix} + \begin{pmatrix} \sqrt{l_1^2 - C^2} \\ -\sqrt{l_1^2 - C^2} \end{pmatrix} \begin{bmatrix} \sin \psi \\ \cos \psi \end{bmatrix}$$

$$\theta_1 = \frac{\left( \frac{l_1^2 - l_2^2 + C^2}{C - 2C} \right) \sin \psi + \sqrt{l_1^2 - C^2} (-\cos \psi)}{\left( \frac{l_1^2 - l_2^2 + C^2}{C - 2C} \right) \cos \psi + \sqrt{l_1^2 - C^2} \sin \psi}$$





# Cinemática directa cadena cerrada



$$l_{2x}^2 = l_2^2 - l_{2y}^2$$

$$l_{2x} = \sqrt{l_2^2 - l_1^2 \sin^2\theta_1}$$

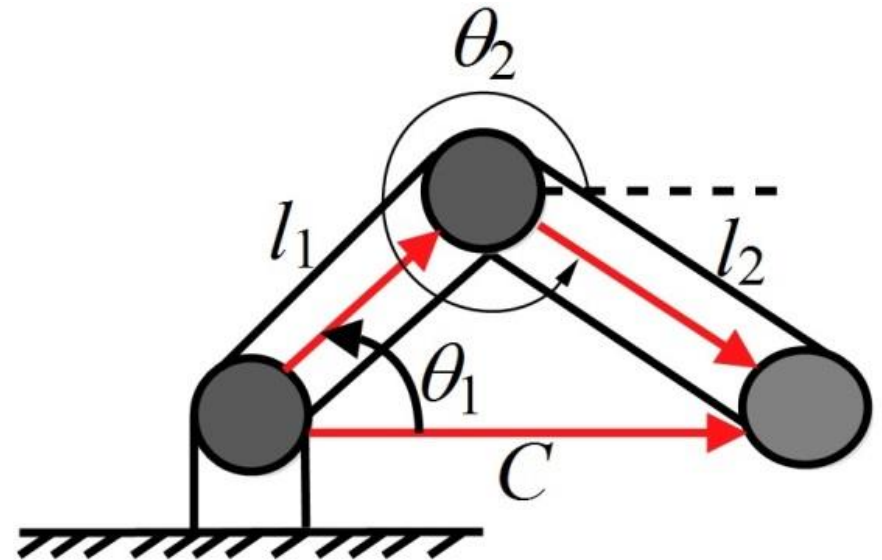


# Cinemática inversa cadena cerrada

- Cinemática inversa cadena cerrada.

$$\theta_1 = \cos^{-1} \left( \frac{C^2 + l_1^2 - l_2^2}{2Cl_1} \right)$$

$$\theta_2 = \cos^{-1} \left( \frac{C^2 + l_2^2 - l_1^2}{2Cl_2} \right)$$







## Dinámica fase simple

$$L(\mathbf{q}, \dot{\mathbf{q}}) = K(\mathbf{q}, \dot{\mathbf{q}}) - U(\mathbf{q})$$

$$K_i(\mathbf{q}, \dot{\mathbf{q}}) = \sum_{i=1}^n \frac{1}{2} m_i \mathbf{v}_i^T \mathbf{v}_i + \frac{1}{2} \boldsymbol{\omega}_i^T I_i \boldsymbol{\omega}_i$$

$$U_i(\mathbf{q}) = \sum_i^n m_i g h_i$$

$$\frac{d}{dt} \left[ \frac{\partial L(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}_i} \right] - \frac{\partial L(\mathbf{q}, \dot{\mathbf{q}})}{\partial \mathbf{q}} = \boldsymbol{\tau}$$



## Dinámica fase simple

$$\tau_1 = \ddot{\theta}_1 (m_1 l_{c1}^2 + m_2 l_1^2 + I_{1zz}) + 2l_1 l c_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_2 + 2l_1 l c_2 \operatorname{sen}(\theta_1 - \theta_2) \dot{\theta}_2^2 + (m_1 l c_1 + m_2 l_1) g \cos \theta_1$$

$$\tau_2 = \ddot{\theta}_2 (m_2 l_{c2}^2 + I_{2zz}) + 2l_1 l c_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_1 - 2l_1 l c_2 \operatorname{sen}(\theta_1 - \theta_2) \dot{\theta}_1^2 + m_2 l c_2 g \cos \theta_2$$



# Dinámica fase simple

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$$

$$M(\mathbf{q}) = \begin{bmatrix} m_1 l_{c1}^2 + m_2 l_1^2 + I_{1zz} & 2l_1 l_{c2} \cos(\theta_1 - \theta_2) \\ 2l_1 l_{c2} \cos(\theta_1 - \theta_2) & m_2 l_{c2}^2 + I_{2zz} \end{bmatrix}$$

$$C(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} 0 & 2l_1 l_{c2} \text{sen}(\theta_1 - \theta_2) \dot{\theta}_2 \\ -2l_1 l_{c2} \text{sen}(\theta_1 - \theta_2) \dot{\theta}_1 & 0 \end{bmatrix}$$

$$\mathbf{g}(\mathbf{q}) = \left[ (m_1 l_{c1} + m_2 l_1) g \cos \theta_1 \quad m_2 l_{c2} g \cos \theta_2 \right]^T$$



# Dinámica fase cerrada

$$\mathbf{q}' = [\theta_1 \quad \theta_2 \quad \theta_3]^T$$

$$\alpha(\mathbf{q}') = \theta_1$$

$$\Theta = [1 \quad 0 \quad 0]^T$$

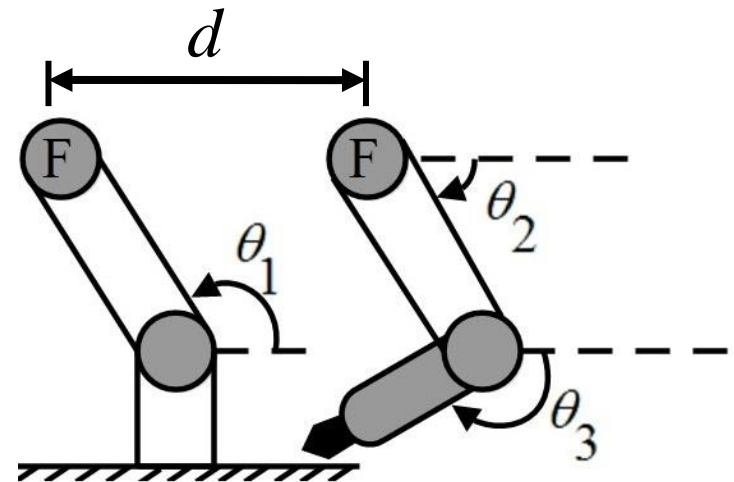
$$\alpha(\mathbf{q}') = \Theta \mathbf{q}'$$

$$\varphi(\mathbf{q}') = \begin{bmatrix} l_1 \cos \theta_1 - l_2 \cos \theta_2 - l_3 \cos \theta_3 - d \\ l_1 \sin \theta_1 - l_2 \sin \theta_2 - l_3 \sin \theta_3 \end{bmatrix}$$

$$\mathbf{R} = [\varphi(\mathbf{q}') \quad \alpha(\mathbf{q}')]^T$$

$$\gamma(\mathbf{q}') = \partial \mathbf{R} / \partial \mathbf{q}'$$

$$\mathbf{R}(\mathbf{q}') = \gamma_{\mathbf{q}'}^{-1}(\mathbf{q}') \begin{bmatrix} \mathbf{0}_{(n'-n) \times n} \\ \mathbf{I}_{n \times n} \end{bmatrix}$$





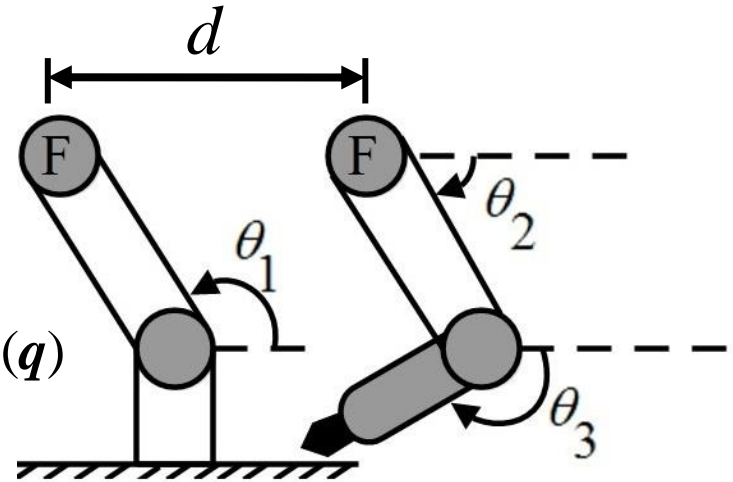
# Dinámica fase cerrada

$$M^*(q') = R(q')^T M(q') R(q')$$

$$C^*(q') = R(q')^T C(q', \dot{q}') R(q') + R(q')^T M(q') \dot{R}(q)$$

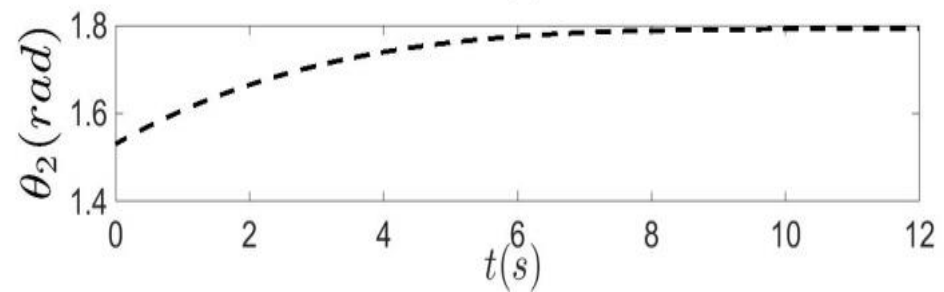
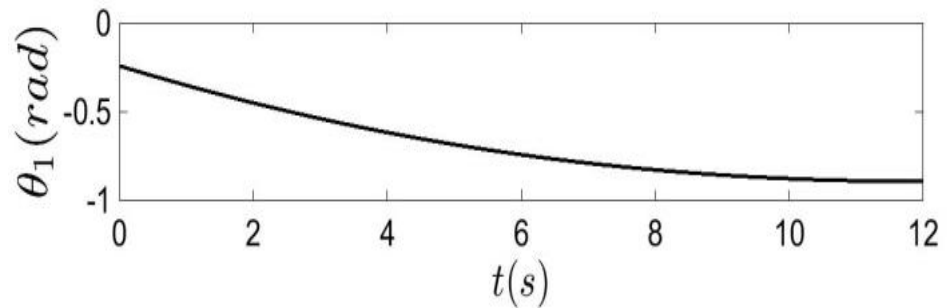
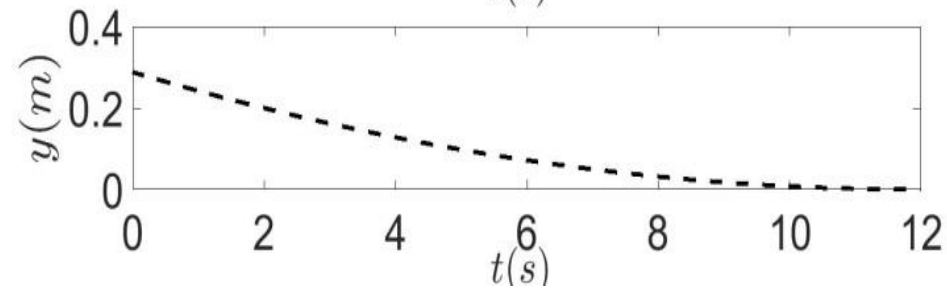
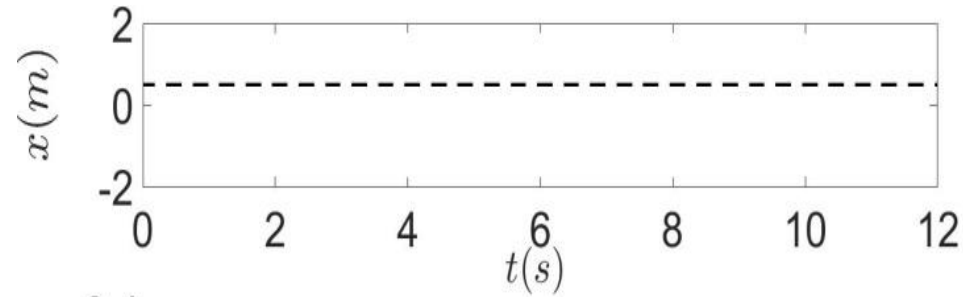
$$g^* = R(q)^T g(q') R(q)$$

$$M^*(q') \ddot{q} + C(q', \dot{q}') \dot{q}' + g(q') = \tau$$



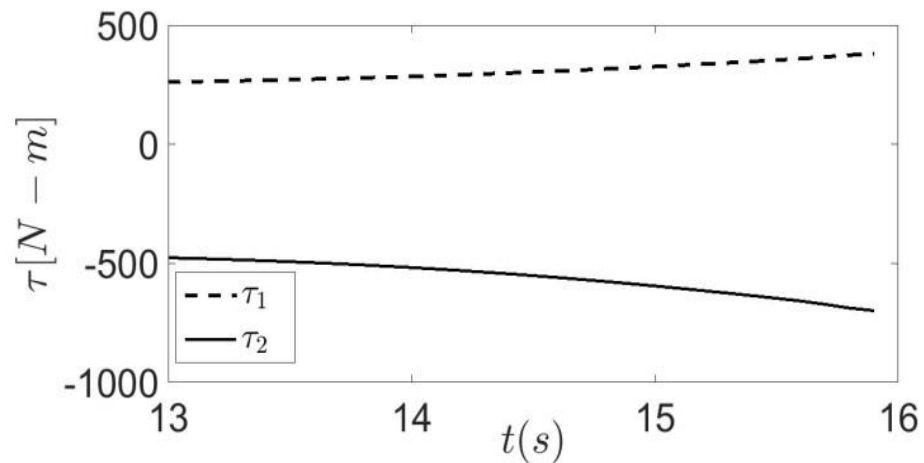
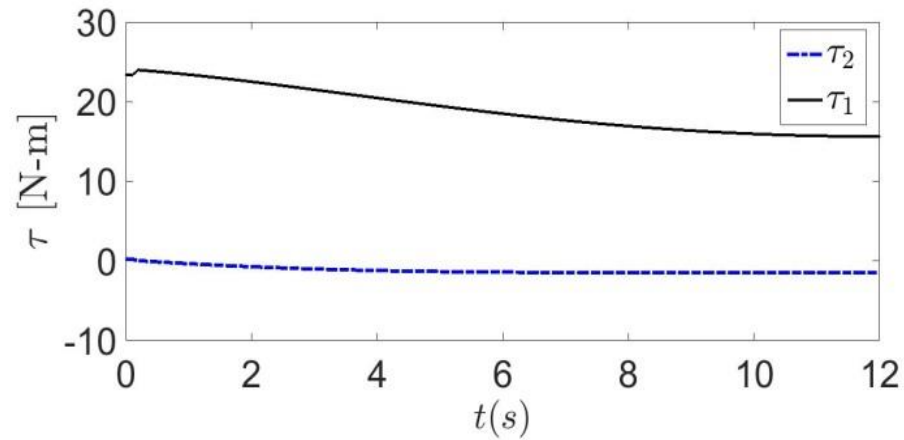


# Simulación y animación.



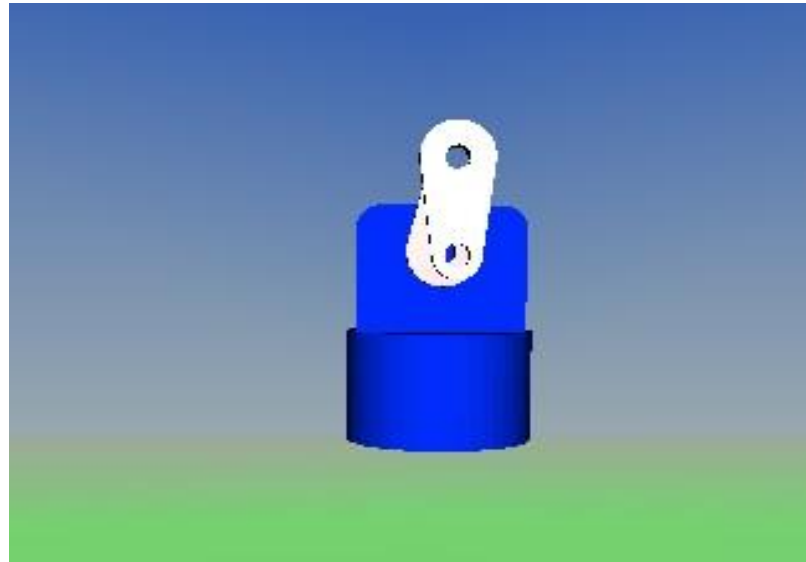


# Simulación y animación.





# Simulación y animación.







# Referencias.

- Lewis FL., Dawson DM, Abdallah CT. (2004). *Robot Manipulator Control: Theory and Practice*. New York, USA: Marcel Dekker Inc.
- Tsai L-W. (1999). *Robot Analysis: the mechanics of serial and parallel manipulators*. New York, USA: Jhon Wiley & Sons, Inc.
- Craig J. (1989). *Introduction to Robotics: mechanics and control*. New York, USA: Addison – Wesley.
- Ollero A. (2001). *Robótica: manipuladores y robots móviles*. Barcelona, España: MARCOMBO.
- Fu K.S., Gonzalez R.C., Lee C.S.G., *Robotics: Control, Sensing, Vision and Intelligence*. México: McGraw-Hill.
- Shigley J.E, Uicker J.J. (1988). *Teoría de máquinas y mecanismos*. Edo. México, México: Mc Graw Hill/Interamericana de México.
- Kelly R., Santibáñez V., Loria A. (2005), *Control of Robot Manipulators in Joint Space*: Prentice Hall, México.



# Referencias.

- Ghorbel, F. H., Chételat O., Gunawardana R., Longchamp R. (2011). Modeling and Set Point of Closed–Chain Mechanisms: Theory and Experiment, *IEEE Transactions on Control systems Technology*, 8(5), 801-815.
- D. Ilia, A., Cammarata, y Sinatra R. (Junio 2007). *A novel formulation of dynamic balancing of five – bar linkages*, Congreso Mundial IFToMM, Besanzón, Francia.
- Soto I., y Campa R. (2011). *Sobre Modelado dinámico de robots paralelos: Mecanismo de cinco barras como caso de estudio*, XIII Congreso Mexicano de Robótica, San Luís Potosí, México.
- Unda J., y García J. (1985). Análisis cinemático y dinámico de sistemas mecánicos formados por varios sólidos rígidos, *Revista internacional de métodos numéricos para cálculo y diseño de ingeniería*, 1(4), 31 – 48.
- Campos C., Campa R., Llama M. y Pámanes A. (2011). Modelling, Simulation and Analysis of a 5-Dof Parallelogram–Link Biped Mechanism, *Praise Worthy Prize S.r.l: International Review on Modelling and Simulations (I.R.E.M.O.S)*, 4(6), 3337 – 3352.
- Díaz S., Gutierrez C., Valdés D., y Hernández B. (2016). Diseño y control en tiempo real de un brazo de robot mediante una interfaz virtual, *Revista Tecnológica e Innovación*, 3(9), 49-60.
- López F., Serna L., y Olguín J. (2016). Desarrollo de un exoesqueleto para la rehabilitación de miembros superiores, *Revista de Tecnología a Innovación*, 3(7), 1-12.
- Mendoza R., Tapia G., Ortega F. y García J.M. (2016). Aplicación web par el control remoto de mecanismos mediante Arduino, *Revista de Tecnología e Innovación*, 3(6), 1 – 7.



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